

Natural Doublet-Triplet Splitting in Supersymmetric SO(10) Models¹

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Abstract

We construct a supersymmetric SO(10) model, where the Dimopoulos-Wilczek mechanism for doublet-triplet splitting is stable under the addition of Higgs superfields belonging to $\mathbf{126} + \overline{\mathbf{126}}$ needed to implement the see-saw mechanism for neutrino masses and where the charged fermion and neutrino mass spectra arise from a single set of $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs representations.

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In discussing supersymmetric grand unified theories, one has to deal with the vexing problem [1] of doublet-triplet splitting. The problem arises from the simultaneous requirement that the $SU(2)_L$ doublet and the color triplet submultiplets of a Higgs multiplet that generates fermion masses must have very disparate masses: the doublet mass must be of order of the electroweak scale v_{wk} to cause $SU(2)_L \times U(1)_Y$ breaking whereas the color triplet mass must be of order of the GUT scale, M_U in order to suppress rapid proton decay. The simplest way to achieve this is to fine tune the parameters of the superpotential. While such a tree level fine tuning is protected by quantum corrections, thanks to the non-renormalization theorem of supersymmetry, it requires unnatural adjustment of parameters and is unlikely to be the path chosen by nature. It is somehow more believable, if underlying group theoretical constraints guarantee the splitting for arbitrary choice of parameters of the theory. This possibility is called natural (or automatic) doublet-triplet splitting. In this letter, we study this question in the framework of realistic $SO(10)$ models. Needless to say that recent indications for neutrino masses in various experiments have made $SO(10)$ a more interesting GUT model to consider. It is therefore timely to address different aspects of these models.

In the minimal $SO(10)$ models, Higgs superfields belonging to **10**-dim. representations of $SO(10)$ are used to generate bulk of the quark and lepton masses. One important prediction of the dominance of **10**-dim. Higgs in generating fermion masses is the equality $m_b = m_\tau$ at GUT scale. When extrapolated to the weak scale, this relation predicts the ratio m_b/m_τ in good agreement with observations. The **10**-dimensional multiplet contains two $SU(2)_L$ -doublets (which we denote by H_u, H_d) and a color triplet and anti-triplet (denoted here as $\xi_1 (\mathbf{3}) + \xi_2 (\bar{\mathbf{3}})$). The problem of doublet-triplet splitting in $SO(10)$ model boils down to understanding why $M_\xi \simeq M_U$ whereas $m_{H_{u,d}} \simeq v_{wk}$. Similar discussions will apply to the **126**-dim. representation too, when it plays a role similar to the **10**-dim. multiplet.

Several years ago, Dimopoulos and Wilczek [2] (DW) suggested a way to solve this problem for **10**-dim. multiplet. They proposed using a **45**-dim. Higgs multiplet (denoted by **A**) to break the $SO(10)$ -symmetry by giving vev only to the (1,1,15) submultiplet of **A** under $SU(2)_L \times SU(2)_R \times SU(4)_C$ subgroup and keeping the (1,3,1) component to have zero vev naturally. Since **A** is anti-symmetric in the $SO(10)$ indices, the implementation of this mechanism requires that there be at least two **10**-Higgs representations (denoted by H_1 and H_2). It is then clear that, if

$$\langle A \rangle = \eta \otimes \text{diag}(p, p, p, 0, 0), \quad (1)$$

where

$$\eta \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

a coupling in the superpotential of the form $\mathbf{A}H_1H_2$, will make all the triplets superheavy while leaving the four doublets H_{1u} , H_{1d} , H_{2u} , and H_{2d} light. The subscripts u and d denote the doublets that can couple to up and down type quarks respectively. Since more than two light doublets are known to effect unification of couplings in an adverse manner, one will have to make one pair of these doublets superheavy. One can solve this problem (a) by letting one of the **10**-Higgs multiplets (say, H_2) not couple to fermions and (b) by giving H_2 a direct superheavy mass in the superpotential, i.e., μH_2H_2 . It has recently been argued [3] that this may not lead to a strong suppression of proton decay though enough to keep the model phenomenologically viable. We will not worry about this in order to keep the model simpler.

When one tries to implement the DW idea in realistic models, one immediately runs into difficulties. To appreciate this, let us first note that, a SUSY SO(10) model must have a **126** + $\overline{\mathbf{126}}$ multiplet pair[4] in order to implement the see-saw mechanism to understand the small neutrino masses [5]. In the presence of the **126** + $\overline{\mathbf{126}}$ pair (denoted by $\Delta + \overline{\Delta}$), one has the coupling $\Delta A \overline{\Delta}$, which induces a vev for the (1,3,1) component of \mathbf{A} , thereby destroying the doublet-triplet splitting. If the coupling is forbidden and the B-L symmetry breaking by $\Delta + \overline{\Delta}$ vev's is accomplished by a term in the superpotential of the form $(\Delta \overline{\Delta} - M^2)S$ (S being a gauge singlet superfield), then there are a large number of light superfields (due to the SU(126) symmetry of the superpotential), that destroy the good unification properties. Something different must therefore be done.

A second requirement that we demand of the minimal SO(10) theory, is inspired by the recent observation [6] that a single **10** and single $\overline{\mathbf{126}}$ coupling to fermion generations can cure both the problem of bad mass relations for the second generation quarks and leptons, i.e., ($m_s = m_\mu$ and $m_d = m_e$ at M_U) and also generate neutrino masses, provided the light Higgs doublets responsible for charged fermion masses is a linear combination of doublets from the **10** and $\overline{\mathbf{126}}$ multiplets. To achieve this mixing, the SO(10) symmetry must be broken by a **210**-dim. Higgs multiplet and that there be no extra light doublets or color triplet fields left over from the Δ or $\overline{\Delta}$ in this process. It turns out that if the superpotential contains only a term of the form $\Phi \Delta H$ (and no $\Phi \overline{\Delta} H$ where Φ and H respectively denote the **210** and **10** dimensional multiplets), this requirement is satisfied. Combinations of **45** + **54** often used (e.g. in Ref. [3]) are not adequate for this purpose. This is a

non-trivial constraint on model building. Below we present a model which satisfies all our requirements, i.e.,

- (a) correct GUT symmetry breaking down to $SU(3)_C \times U(1)_{em}$,
- (b) only one pair of light Higgs doublets obtained naturally due to the existence of a symmetry (perhaps softly broken),
- (c) the light doublets (both H_u and H_d types) are linear combinations of the doublets in the **10** and $\overline{\mathbf{126}}$ multiplets.

The Model: We consider the local symmetry group of the model to be $SO(10)$, with an additional global symmetry $U(1)_{PQ} \times Z_{16}^H$ with both symmetries softly broken by some dimension two terms in the superpotential.

We will demand the symmetry to be respected only by dimension three and higher terms of the superpotential and not by the soft dimension two terms. Thus, there is no light axion in the model. The following set of Higgs multiplets are chosen for the model: $\Phi_1(210)$, $\Phi_2(210)$, $A(45)$, $\Delta(126)$, $\overline{\Delta}(\overline{126})$, $H_{1,2}(10)$. Their transformation properties under the global symmetry groups are given in Table 1. (We denote the sixteenth root of unity by z .)

The gauge invariant superpotential can be written as a sum of three terms:

$$W = W_m + W_H^{(3)} + W_H^{(2)}, \quad (2)$$

where

$$W_H^{(3)} = \lambda_1 \Phi_1^2 \Phi_2 + \lambda_2 \Phi_2 \Delta \overline{\Delta} + \lambda_3 \Phi_1 A A + \lambda_4 A H_1 H_2 + \lambda_5 \Phi_1 \Delta H_1, \quad (3)$$

$$W_H^{(2)} = \mu_1 \Phi_1^2 + \mu_2 \Phi_2^2 + \mu_3 \Delta \overline{\Delta} + \mu_4 A A + \mu_5 H_2 H_2 + \mu_6 \Phi_1 \Phi_2, \quad (4)$$

$$W_m = h_{ab} \Psi_a \Psi_b H_1. \quad (5)$$

We choose all main parameters μ_i to be of order of the GUT scale M_U . The two terms $W_H^{(3)} + W_H^{(2)}$ lead to a number of degenerate supersymmetric minima, one of which has the desired pattern of symmetry breaking with $\langle A \rangle$ given in Eq. (1) naturally with the vev of Φ_1 only along the (1,1,1) and (1,1,15) directions and naturally with that of Φ_2 along the (1,1,1), (1,1,15), and (1,3,15) directions. By the Dimopoulos-Wilczek mechanism, this leads to natural doublet-triplet splitting for both H_1 and H_2 ; the μ_5 term makes one pair of the light doublets superheavy as required phenomenologically.

To show that this pattern of vev's preserves supersymmetry down to the electroweak scale, we call $\langle \Phi_i(1, 1, 1) \rangle = a_i$, $\langle \Phi_i(1, 1, 15) \rangle = b_i$, $\langle \Phi_i(1, 3, 15) \rangle = c_i$, $\langle A(1, 1, 15) \rangle = p$, $\langle A(1, 3, 1) \rangle = q$, $\langle \Delta(1, 3, \overline{10}) \rangle = \langle \overline{\Delta}(1, 3, 10) \rangle = v_R$. All these vev's are of order M_U . Let us first write down the vanishing F-term conditions

for the $\langle A \rangle$; we get

$$\begin{aligned}
2\mu_4 p + 2\lambda_3 \frac{\sqrt{2}}{3} p b_1 + 2\lambda_3 \frac{1}{\sqrt{6}} q c_1 &= 0, \\
2\mu_4 q + 2\lambda_3 \frac{1}{\sqrt{6}} q a_1 + 2\lambda_3 \frac{1}{\sqrt{6}} p c_1 &= 0.
\end{aligned} \tag{6}$$

From this, we first see that there is a solution of Eq. (6) for which $p \neq 0$ and $q = 0$ if $c_1 = 0$. This is the vacuum we will focus on and see if the rest of the F-term conditions are satisfied for arbitrary values of the parameters in the superpotential. To see this, let us write down [7] those $F = 0$ conditions with $c_1 = 0$ and $q = 0$.

$$\begin{aligned}
0 &= 2\mu_1 a_1 + \mu_6 a_2, \\
0 &= 2\mu_1 b_1 + \mu_6 b_2 + 2\lambda_1 \frac{b_1 b_2}{9\sqrt{2}} + \lambda_3 \frac{\sqrt{2}}{3} p^2, \\
0 &= \mu_6 c_2 + 2\lambda_1 \left(\frac{a_1 c_2}{6\sqrt{6}} + \frac{b_1 c_2}{9\sqrt{2}} \right).
\end{aligned} \tag{7}$$

$$\begin{aligned}
0 &= 2\mu_2 a_2 + \mu_6 a_1 + \lambda_2 \frac{v_R^2}{10\sqrt{6}}, \\
0 &= 2\mu_2 b_2 + \mu_6 b_1 + \lambda_1 \frac{b_1^2}{9\sqrt{2}} + \lambda_2 \frac{v_R^2}{10\sqrt{2}}, \\
0 &= 2\mu_2 c_2 + \lambda_2 \frac{v_R^2}{10}.
\end{aligned} \tag{8}$$

$$0 = 2\mu_2 v_R + \lambda_2 \left(\frac{a_2 v_R}{10\sqrt{6}} + \frac{b_2 v_R}{10\sqrt{2}} + \frac{c_2 v_R}{10} \right). \tag{9}$$

Eqs. (7)-(9) arise from the F-terms corresponding to Φ_1 , Φ_2 , and Δ (or $\overline{\Delta}$), respectively. It is easy to see that this has non-trivial solutions for all the vacuum expectation values for arbitrary choice of the parameters of the model. This establishes the naturalness of the doublet-triplet splitting.

Let us now write down the doublet Higgsino mass matrix and isolate the massless pair of the Higgs doublets, that generate electroweak symmetry breaking. First we note that the doublets in H_2 pick up superheavy mass due to the μ_5 term and are completely decoupled from the other doublets. Denoting the rest of the

doublets by $(\Phi_{2u}, \Phi_{1u}, \Delta_u, \overline{\Delta}_u, H_{1u})$ and $(\Phi_{2d}, \Phi_{1d}, \overline{\Delta}_d, \Delta_d, H_{1d})$, we can write down their mass matrix in the basis where the above sets of fields denote the columns and rows respectively.

$$\begin{pmatrix} \mu_2 & \tilde{\mu}_6 & 0 & \lambda_2 v_R & 0 \\ \tilde{\mu}_6 & \tilde{\mu}_1 & 0 & 0 & \lambda_5 v_R \\ 0 & 0 & \tilde{\mu}_3 & 0 & 0 \\ \lambda_2 v_R & 0 & 0 & \tilde{\mu}_3 & \lambda_5 v_U \\ 0 & 0 & \lambda_5 v_U & 0 & 0 \end{pmatrix}. \quad (10)$$

Analyzing this matrix, we conclude that it has two zero eigenstates given by:

$$\begin{aligned} H_u &= x_1 H_{1u} + x_2 \overline{\Delta}_u + x_3 \Phi_{1u} + x_4 \Phi_{2u}, \\ H_d &= y_1 H_{1d} + y_2 \overline{\Delta}_d. \end{aligned} \quad (11)$$

We therefore see that the light doublets have the desired property to solve the charged fermion mass puzzles of the simplest SO(10) models following Ref. [6]. All colored triplets in this model are superheavy - thus, proton decay is suppressed.

Let us now turn to the fermion sector. Note that W_m in Eq. (5) has two deficiencies: (a) it does not give correct relation between m_s and m_μ , (b) the absence of the $\overline{\Delta}$ coupling makes the right-handed neutrinos massless. Both these problems are cured when we include the Z_{16}^H -invariant Planck scale induced dimension 4 terms in the superpotential, i.e.,

$$W_m^{(1)} = \frac{1}{M_{Pl}} f_{ab} \Psi_a \Psi_b \Phi \overline{\Delta}. \quad (12)$$

A priori, there are three independent couplings, where the $\Psi\Psi$ bilinear transforms like **10**, **120**, **126**; however, the fermion masses get contribution only from **10**, **126** type couplings. The **10** type couplings simply redefine the original **10** couplings, whereas the **126** type couplings introduce the same structure to the fermion mass as in Ref. [6] and predictions of neutrino masses given in that reference carry over. If f_{ab} are chosen to be order 1 to 3 and the $\overline{\Delta}$ -vev's along the (2, 2, 15) direction are chosen to be of order 100 GeV, the contribution of $W_m^{(1)}$ to fermion masses have the correct order of magnitude. Secondly, the right-handed neutrinos acquire a mass of order $\simeq 10^{12}$ GeV or so if $v_R \simeq 10^{-1} v_U$. This scale for the right-handed neutrinos is helpful in understanding baryogenesis of the universe [9].

In summary, we have constructed an SO(10) model where the Dimopoulos-Wilczek mechanism for doublet-triplet splitting is unaffected by the addition of

$126 + \overline{126}$ multiplets needed for the see-saw mechanism for neutrino masses. We have also been able to show that once the lowest order Planck scale corrections are included, the light Higgs doublets have the correct structure to reproduce the desired quark-lepton mass spectra and lead to predictions for neutrino masses as in Ref. [6]. There are no undesirable light particles in the theory. We hope to have made clear the nontrivial nature of the constraints one would like to impose on a desirable SO(10) model and that by choosing appropriate symmetries, models satisfying all these constraints can indeed be constructed. Perhaps, the existence of one such model demonstrated in this letter will stimulate further thinking to construct simpler models that may yield further insight into SO(10) grandunified theories.

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Table Caption

Table 1: Global symmetry quantum numbers for different fields. $z = e^{i\pi/8}$.

Supermultiplets	$U(1)_{PQ}$ charge	Z_{16}^H
Matter multiplets		
Ψ_3	+1	z^7
Ψ_2	+1	z^7
Ψ_1	+1	z^7
Higgs multiplets		
Φ_1	0	z^{12}
Φ_2	0	z^8
A	0	z^2
Δ	+2	z^2
$\overline{\Delta}$	-2	z^6
H_1	-2	z^2
H_2	+2	z^{12}

Table 1